LS-88
W. Chou
October 1988

Emittance Growth due to Ground Motions

To evaluate the dynamic emittance growth due to ground motions for a synchrotron light source, a method using the so-called transfer function is suggested in Ref. 1. This method assumes, among others, that the displacement of the magnetic elements is equal to that of the ground. In other words, the support effects are neglected. Ref. 2, on the other hand, proposes a normal-mode method to calculate the displacement magnification due to supports. It is then natural to combine the two methods together to get a more complete picture of the vibration - emittance growth problem.

For a single vibration frequency f of the ground, the dynamic emittance growth, $\Delta\varepsilon_d(f)$, can be obtained from a set of equations:

Horizontal:

$$\left(\frac{\Delta\varepsilon_d(f)}{\varepsilon}\right)_x = 2\,\frac{\bar{x}(f)}{\sigma_x} + \left(\frac{\bar{x}(f)}{\sigma_x}\right)^2, \tag{1a}$$

$$\bar{x}(f) = A_x(f) \cdot \delta_m(f)\ , \tag{1b}$$

$$\delta_m(f) = A_m\ (f) \cdot \delta_g(f)\ ; \tag{1c}$$

Vertical:

$$\left(\frac{\Delta\varepsilon_d(f)}{\varepsilon}\right)_y = 2\,\frac{\bar{y}(f)}{\sigma_y} + \left(\frac{\bar{y}(f)}{\sigma_y}\right)^2\ , \tag{2a}$$

$$\bar{y}(f) = A_y(f) \cdot \delta_m(f)\ , \tag{2b}$$

$$\delta_m(f) = A_m(f) \cdot \delta_g(f)\ , \tag{2c}$$

where

ε_x (ε_y) : horizontal (vertical) natural emittance;

$\bar{x}$ $(\bar{y})$: magnitude of the oscillating horizontal (vertical) closed orbit;

σ_x (σ_y) : horizontal (vertical) beam size;

δ_m : magnitude of the vibration of the magnetic elements;

δ_g : magnitude of the vibration of the ground;

A_x (A_y) : horizontal (vertical) closed orbit magnification factor;

A_m : vibration magnification factor. In Ref. 1, A_m is simply set to 1.

For a continuous spectrum of ground vibrations, the total emittance growth is just the integral of $\frac{\Delta\varepsilon_d(f)}{\varepsilon}$ over the frequency region.

Ref. 1 gives an approximate formula for calculating A_x and A_y.

$$A_x(f) = \sum_{s=-\infty}^{\infty} \frac{\nu_x^2}{\nu_x^2 - s^2} \cdot j^s \cdot J'_s\left(\frac{2\pi fR}{v}\right) , \tag{3a}$$

$$A_y(f) = \sum_{s=-\infty}^{\infty} \frac{\nu_y^2}{\nu_y^2 - s^2} \cdot j^s \cdot J_s\left(\frac{2\pi fR}{v}\right) , \tag{3b}$$

in which ν_x (ν_y) is the horizontal (vertical) tune, R the radius of the storage ring, v the propagation velocity of sound waves in the ground, j the square root of -1, and J_s and J_s' the s^{th} order Bessel function and its derivative, respectively. As an example, Fig. 1 shows the dynamic emittance growth $\Delta\varepsilon_d$ in the insertion-device (ID) region of the APS when δ_m is set equal to 1 μm, and the support effects are neglected. The magnification factors A_x and A_y are listed in Table 1. The data we use in the above calculations are as follows:

$$\nu_x = 35.2159,$$
$$\nu_y = 14.2984,$$
$$R = 168.7 \text{ m},$$
$$v = 2500 \text{ m/s},$$
$$\varepsilon_x = 7.937 \times 10^{-9} \text{ m·rad},$$
$$\varepsilon_y = 0.1\ \varepsilon_x,$$
$$\beta_x = 13 \text{ m (in the ID region)},$$
$$\beta_y = 10 \text{ m (in the ID region)},$$

$$\sigma_x = \sqrt{\beta_x\ \varepsilon_x} = 321\ \mu\text{m (in the ID region)},$$

$$\sigma_y = \sqrt{\beta_y\ \varepsilon_y} = 89\ \mu\text{m (in the ID region)},$$

and the Bessel functions are calculated up to order 43.

Table 1

The Closed Orbit Magnification Factors†

Frequency	A_x	Frequency	A_y
f < 62 Hz	1	f < 18 Hz	1
62 < f < 110 Hz	$1 < A_x < 17.9$	18 < f < 38 Hz	$1 < A_y < 15.6$
f > 110 Hz	~17.9	f > 38 Hz	~15.6

†Propagation velocity of sound wave in the ground is assumed to be 2500 m/s.

The formula for computing $A_m(f)$ is given in Ref. 2 for a model consisting of a uniform girder supported by two elastic pedestals (which is a simplification of the magnet-pedestal systems in the APS).

$$A_m(f) = \max_x \left\{ \sum_{p=1}^{\infty} Y^{(p)}(x) \cdot [Y^{(p)}(b) + Y^{(p)}(c) \cdot e^{i\theta_p}] \cdot \frac{k}{\rho(2\pi f_p)^2} M^{(p)} e^{i\phi_p} \right\} \quad (4)$$

in which

$Y^{(p)}(x)$ and f_p	are the normalized p^{th} normal mode and the p^{th} natural frequency of the magnet-pedestal system;
$Y^{(p)}(b)$ and $Y^{(p)}(c)$	are the values of $Y^{(p)}(x)$ at the supporting points b and c (See Fig. 1 of Ref. 2), respectively;
θ_p	is the phase difference between the ground vibrations at two supporting points b and c;
k	is the stiffness of the pedestal;
ρ	is the linear mass density of the girder;
$M^{(p)}$	is usually called the p^{th} dynamic amplification factor and is equal to $1 / \{[1-(\frac{f}{f_p})^2]^2 + (2\zeta_p \frac{f}{f_p})^2\}^{1/2}$;
ϕ_p	is the p^{th} phase angle and is equal to $\tan^{-1} \{2\zeta_p \frac{f}{f_p}/[1-(\frac{f}{f_p})^2]\}$;
ζ_p	is the p^{th} damping factor, which determines the height and the width of the p^{th} resonant peak.

The two quantities $M^{(p)}$ and ϕ_p have universal forms and are reproduced here in Fig. 2 as a convenient reference. One sees that when f is much less than the p^{th} natural frequency f_p, we have $M^{(p)} \approx 1$ and $\phi_p \approx 0$; whereas when $f \gg f_p$, we have $M^{(p)} \approx 0$ and $\phi_p \approx \pi$.

Ref. 2 also shows how to calculate the normal modes for given boundary conditions. The numerical example given there is for the dipole system. Since the dynamic emittance growth is mainly due to the quadrupole vibrations, we now calculate the normal modes of the long girder that supports four quadrupoles and three sextupoles in the dispersive straight section of the APS. The data in use are as follows:

M (mass) = 8000 lb (total 16,000 lb supported by two girders)

L (length) = 204 in

ℓ_1 (distance between the end pts. and the supporting pts.) = 45.5 in

E (Young's Modulus) = 29×10^6 lb/in^2

I (moment of inertia) = 190 in^4

k (stiffness of pedestal) = 10×10^5 lb/in

Figures 3(a) - 3(e) show the first five modes of the long girder. The vibration magnification factor A_m is shown in Fig. 4, when the term $e^{i\theta_p}$ in Eq. (4) is close to 1 (this can be justified, insofar as the half-wavelength of the ground vibration is much larger than the distance between the two supporting points). Some of the values of A_m are listed in Table 2. More resonant peaks are expected if higher order (>5) modes are taken into account.

Table 2

The Vibration Magnification Factor

Frequency	A_m
$f < f3$	3.5
$f \sim f3$	first resonant peak
$f3 < f < f5$	$\lesssim 1$
$f \sim f5$	second resonant peak
$f > f5$	$\rightarrow 0$

Note: f3 = 40.98 Hz, f5 = 128.29 Hz. The first, second and fourth modes have no contributions to A_m if the phase difference θ_p is much less than π.

By combining Eqs. (1) - (4), one can estimate the allowed vibration magnitudes of the ground for a given emittance growth, say 10%; these are shown in Fig. 5. The rising tails in the high-frequency region of Fig. 5 are somewhat deceptive--they would go up and down repeatedly if more higher order modes were taken into consideration. Since the low-frequency region is our main concern, the lower left corners of both graphs in Fig. 5 are enlarged in Fig. 6. One can see clearly that at the third and fifth natural frequencies of the long girder system, resonances drive the allowed vibration magnitude of the ground down to virtually zero. As a comparison, Fig. 7 shows the allowed vibration magnitudes of the ground when support effects are ignored.

If the measured propagation velocity v differs from the assumed value (2500 m/s in the above calculations), the f values in Table 1, as well as in Figs. 1 and 7, should be scaled by using f/v = constant for different v's (this is obvious in view of Eqs. 3(a) and 3(b)), whereas the values of those f's in Fig. 3 are functions of the mechanical structure of the magnet-support system only and are independent of v. Thus, some caution is needed in obtaining Fig. 5 or 6 for different v's. One should also note that a possible way to shift the resonant frequencies in Fig. 5 or 6, when it is necessary, is to change some mechanical parameter of the magnet system (for instance, the moment of inertia of the girder).

References

1. T. Aniel and J. L. Laclare, "Sensitivity of the ESRP Machine to Ground Movement", ESRF-SITE-86-04.

2. W. Chou, ANL Light Source Note LS-77 and Addendum LS-77A (1987).

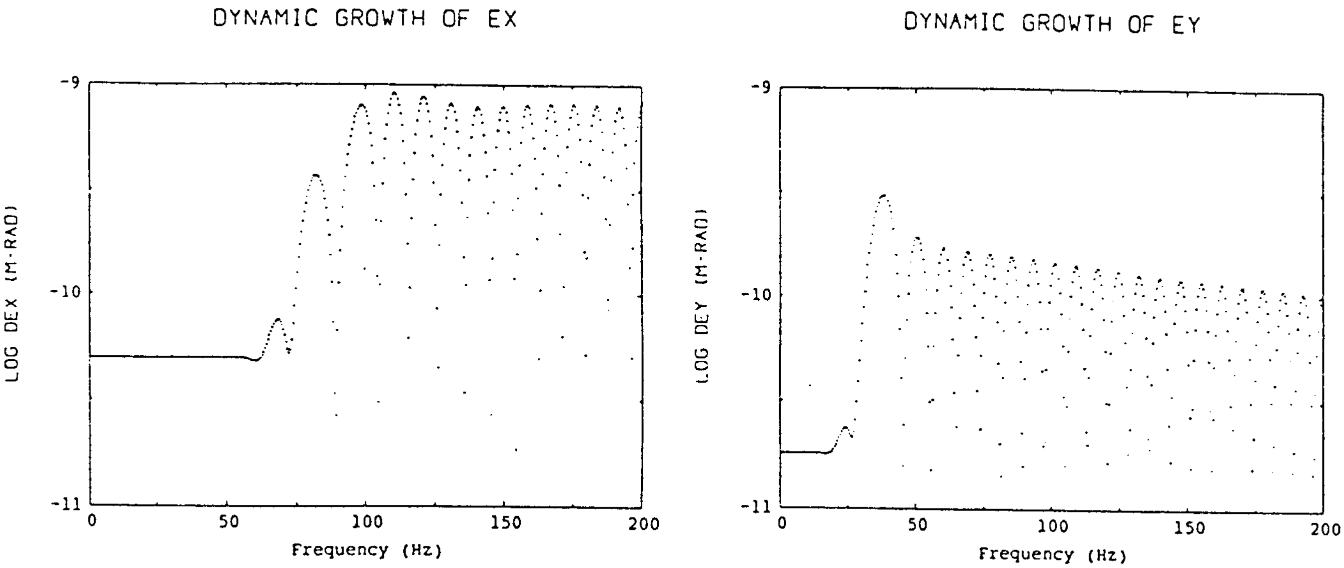


Fig. 1. The dynamic emittance growth $\Delta\xi_d$ in the ID region of the APS when the magnitude of the ground vibration is 1 μm. The support effects are neglected.

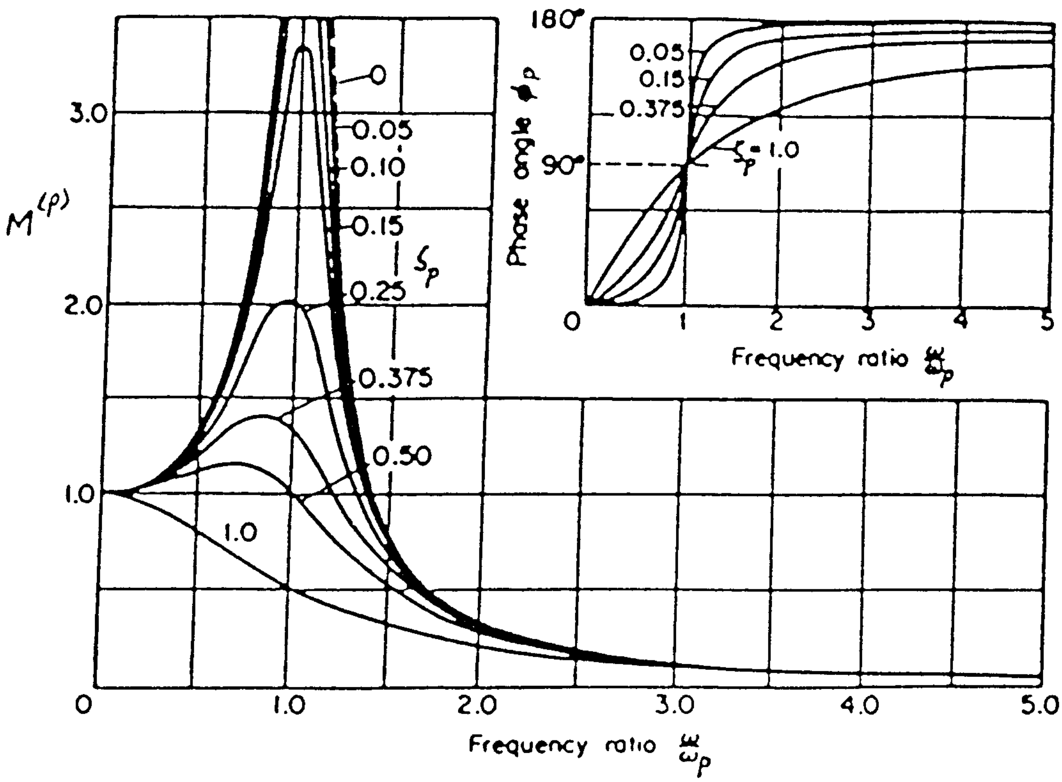


Fig. 2. The dynamic amplification factor M and the phase angle ϕ of the p^{th} mode.

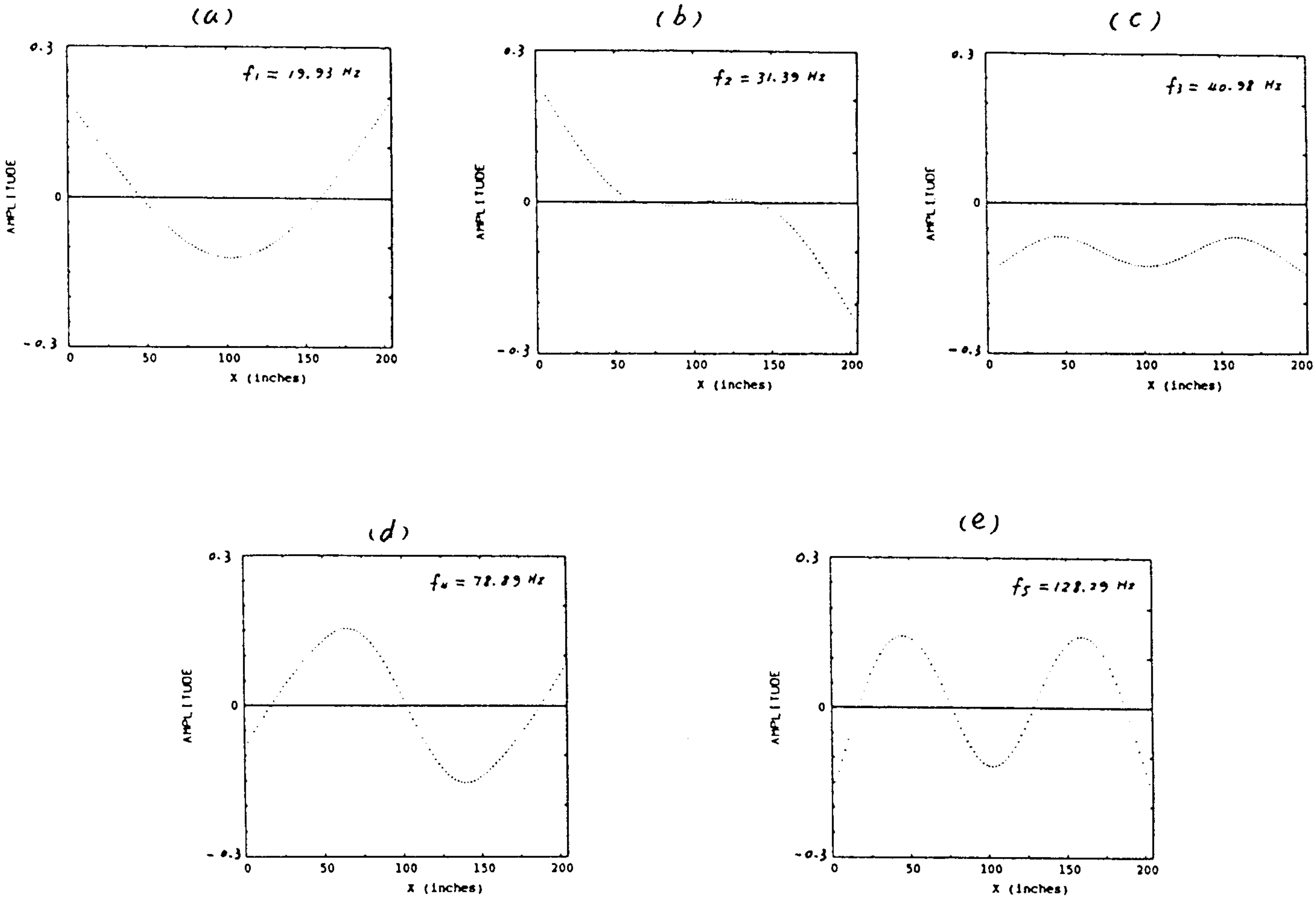


Fig. 3. The first five normal modes of the long girder in the dispersive straight section of the APS.

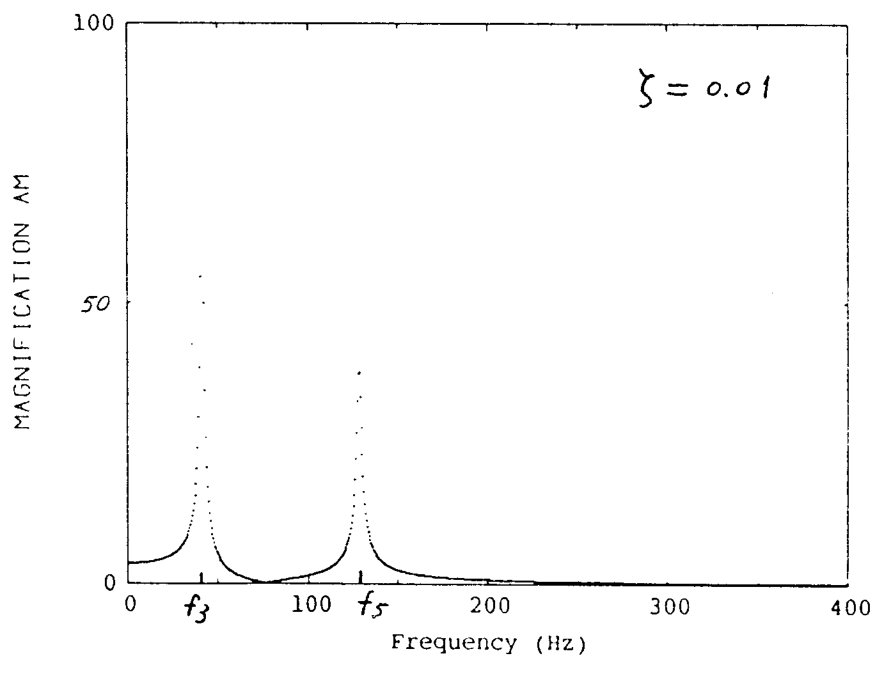
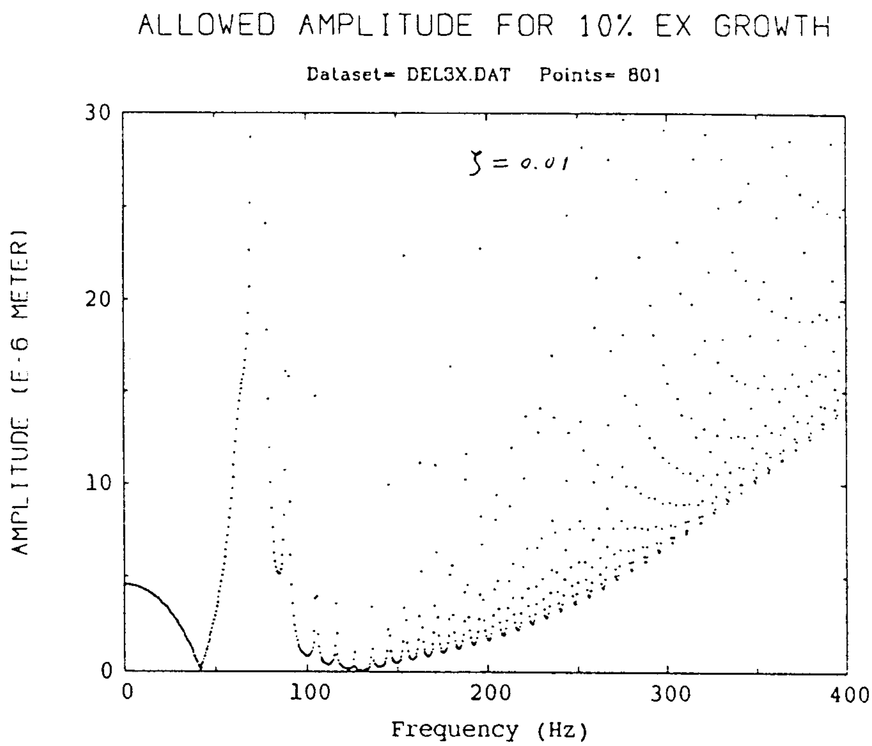


Fig. 4. The vibration magnification factor A_m of the long girder system.



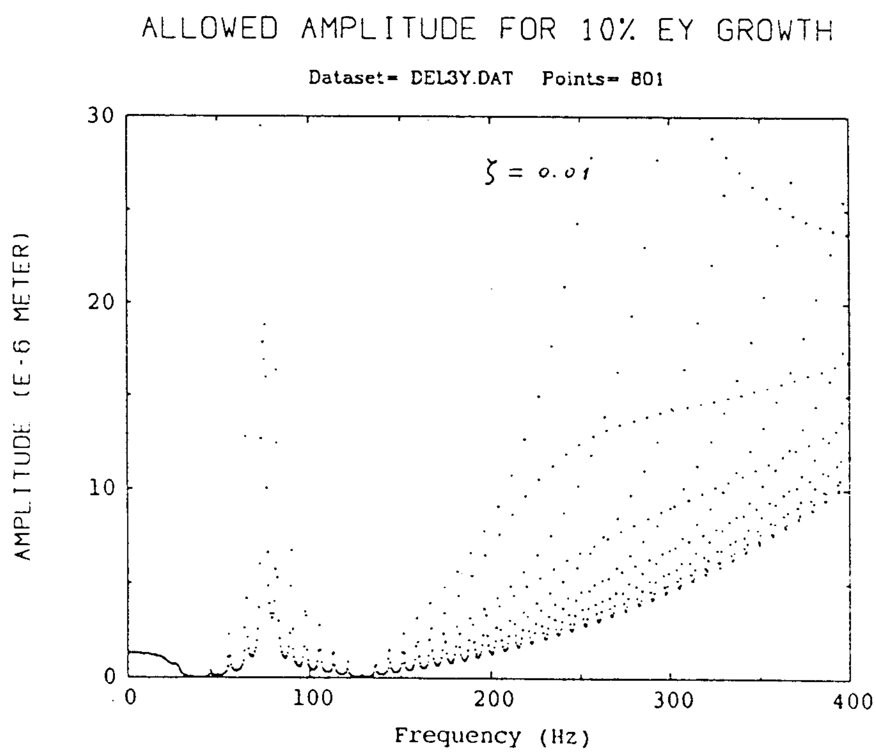


Fig. 5. The allowed vibration magnitude of the ground when the emittance growth in the insertion device region is 10%. The rising tails are deceptive (see text).

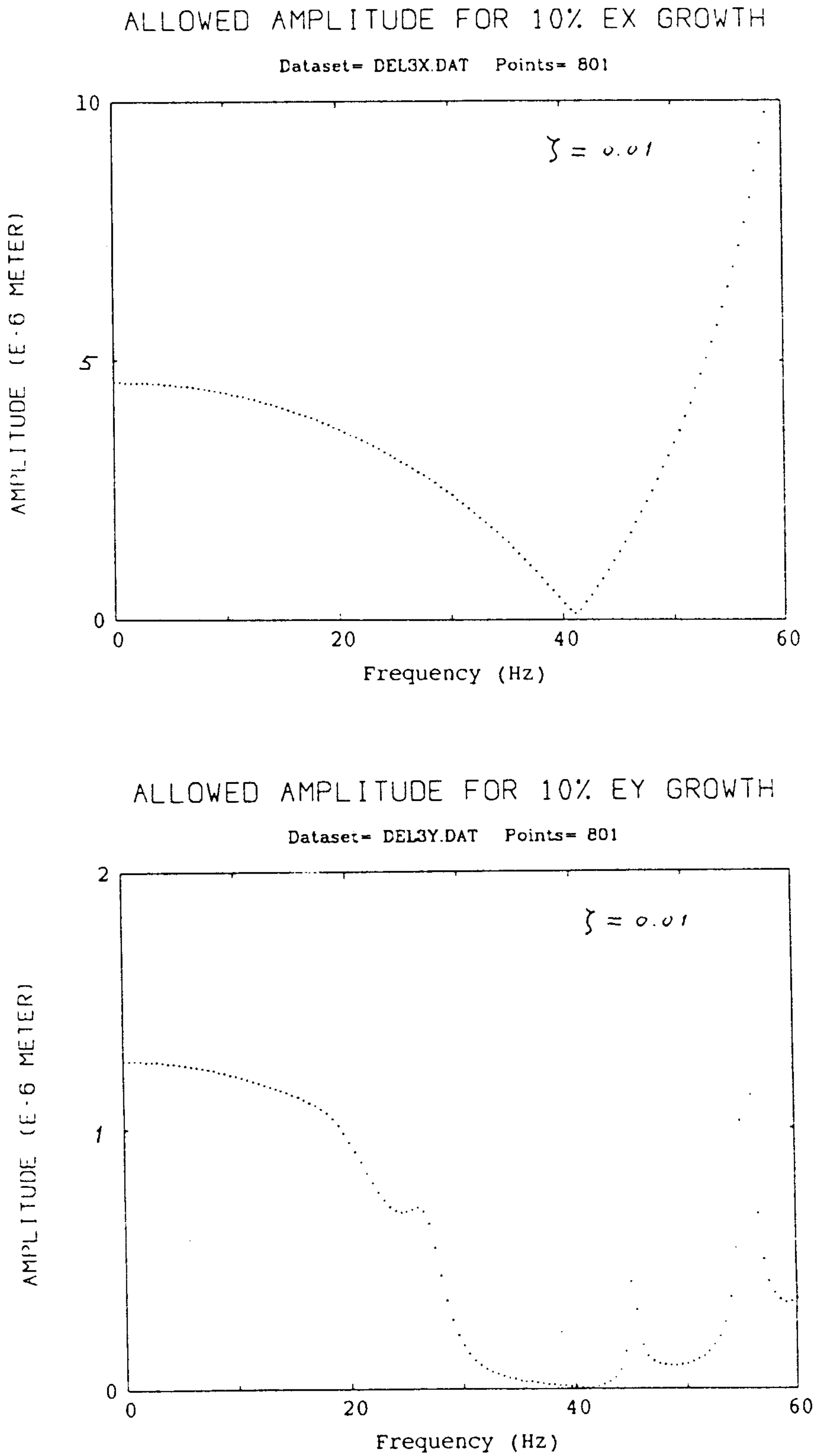
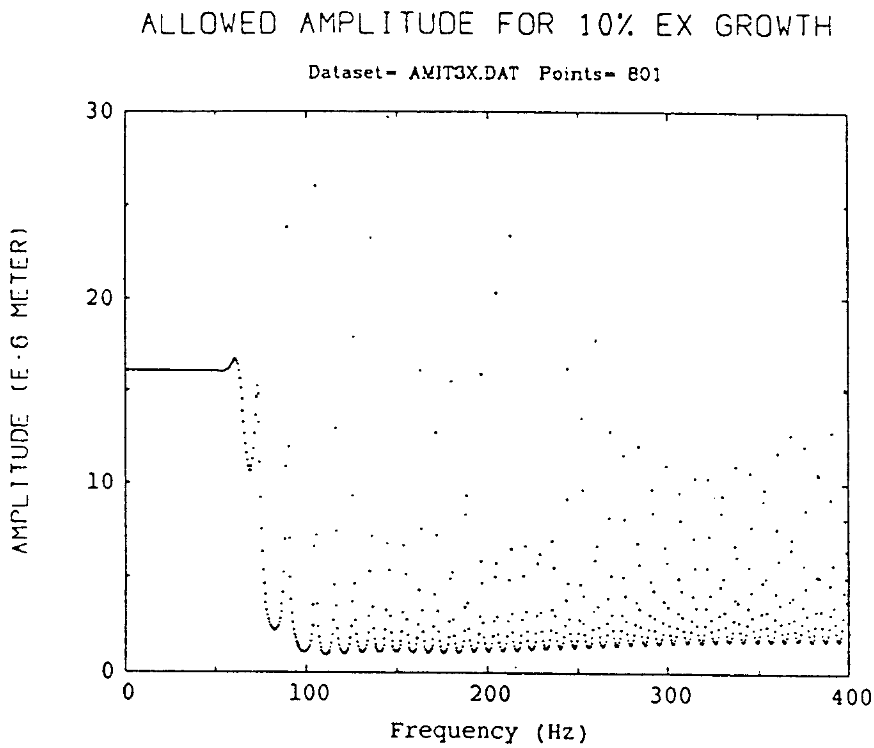


Fig. 6. The enlarged lower left corners of Fig. 5.



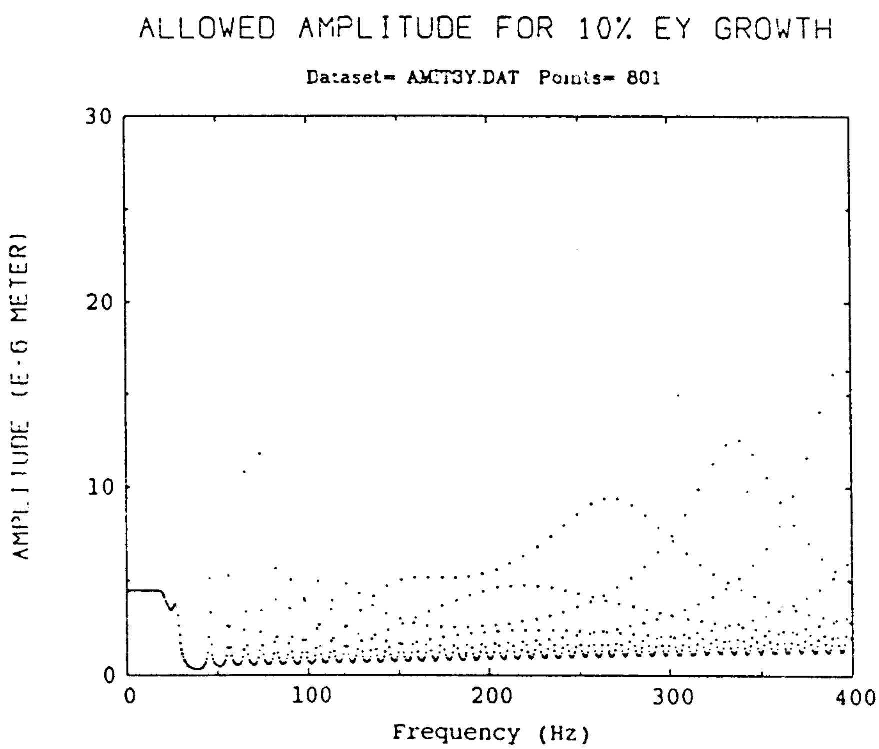


Fig. 7. The same as in Fig. 5, except that the support effects are now ignored.